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Computational Blood Cell Mechanics
Road Towards Models and Biomedical Applications

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the computational modeling and its power.
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Preface

Biology is becoming more and more of a quantitative science, some even say that it is at a stage similar to physics pre-Newton [16]. It is waiting for breakthroughs not only in terms of biological discoveries but also in terms of new methods that will catalyse them, similar to Newton’s mechanics being catalysed by the development of calculus.

At the same time, experiments using computer simulations have become very accessible and in some cases even preferable to biological experiments. The computational resources today are fairly easily available and can be used to test various hypotheses before diving into more complicated or expensive biological testing procedures.

These two trends are steadily converging and attract considerable attention in both the biomedical and the computational fields. How did we find ourselves in the middle of it?

Both of us are trained mathematicians. We have studied numerical analysis, optimisation and mathematical modeling. Our paths had briefly crossed at the Comenius University in Bratislava, Slovakia, where we both got our master’s degrees in Mathematics but then led us on to separate trails. Ivan to Ghent University, Belgium and Fachhochshule St. Poelten, Austria and Iveta to New Jersey Institute of Technology, Newark, USA. During this time, we both somehow intuitively navigated towards more applied topics and when we met again at University of Žilina it was on a project to model red blood cells in microfluidic devices.

Looking back, the beginnings of the project seem more like a random walk than a steady progress towards a good model. Two steps forward, one step back. And then another to a dead-end sidetrack. Yes, research is like that but talking together some time later we both agreed that it could have been easier. Had we known some more basics about membrane biomechanics and about building cell models, we could have saved ourselves quite a bit of time and frustration.

With that hope we are writing this book. We would like to help young researchers entering this promising field and professionals who could use blood cell models in their applied work get oriented and started. We focus on cell mechanics and specifically red blood cell mechanics, even though we briefly touch upon other types of blood cells and circulating tumor cells. Computational blood cell models are useful in many applications such as diagnostics.
of diseases using various lab-on-a-chip concepts, monitoring of response to treatment and also in primary research of blood flow and its properties.

Red blood cells perform several biological functions and their abilities directly depend on their shape and structural stiffness. The cells respond to external mechanical stimuli and interact with their surroundings by changing their shapes or mechanical properties, which may even lead to their damage under extreme conditions. Cell models have to account for complex processes that happen at varied length scales - from molecular to microscale, at which the cells live, and still conform to the macro behavior of cell flow. That is not an easy task and we would like if this book could help in tackling it.

Some things this book is not. It is not a universal book on building models in general. While certainly invaluable, that kind of book would be extremely difficult to write and quite difficult to use once one wants to build a specific model, e.g., a model of a cell. We try to point out some general principles of building good models, but more often than not, the principles we mention apply specifically to cell models. Also this book is not an exhaustive treatment of all computational cell models out there. We try to set things into context and perspective by giving the reader some background, but our main focus is to help a modeler new to the field of blood cell modeling overcome initial hurdles and distill what is essential. We do not cover biological or chemical processes inside the cell, but rather remain on the whole cell scale.

We hope that the book will help the readers see models as bridges between different levels of understanding and that it will inspire new work in the exciting and very promising field of cell modeling.

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